GCSE REVISION GUIDE GRADE 4

Number

Reciprocal

Means 1/x
The reciprocal of 4 is 1 over 4 which is ¼ = 0.25

Decimals

½	0.5
¼	0.25
¾	0.75
⅓	0.3
⅔	0.6
⅕	0.2
$\frac{1}{10}$	0.1

Product of Primes

'Tree'
Product – multiplication
Prime – number that only divides by 1 and itself
Prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, etc.
To write 36 as a product of primes break 36 down by dividing:

36
|
2 x 18
|
2 x 9
|
3 x 3

As a product of primes: 2 x 2 x 3 x 3
As a product of primes in index notation: $2^2 \times 3^2$

LCM and HCF

LCM – Least/Lowest Common Multiple (of 2 numbers)
Write out the times table for each number and find the lowest that appears in each
Find the LCM of 10 and 15
10 – 10, 20, 30
15 – 15, 30

HCF – Highest Common Factor
Write down the factors for each number (factor is a number that goes into another number exactly – 1 and the number will always be a factor) and find the highest number that occurs in each.
The highest common factor of 36 and 45:
36 – 1, 2, 3, 4, 6, 9, 12, 18, 36
45 – 1, 3, 5, 9, 15, 45

Multiplication and division of powers

When 2 letters or numbers with powers (which are the same) are multiplied then the powers are **added**
Simplify: $y^2 \times y^3 = y^5$

When 2 letters or numbers with powers (which are the same) are divided then the powers are **subtracted**
Simplify $c^5 \div c^3 = c^8$

Multiply and divide by negative numbers

Any number without a sign is positive
When you multiply or divide with positive and negative numbers you must combine the signs
+ + = + - - = + + - = - - + = -

2 x 3 = 6 -2 x 3 = -6 2 x -3 = -6 -2 x -3 = 6

12 ÷4 = 3 -12 ÷ 4 = -3 12 ÷ -4 = -3 -12 ÷-3 = 4

Rounding

Decimal places – Has to have this number of numbers after the decimal point but you must consider the next number and if the number is 5 or more, the previous number moves up by 1.

- Round 2.43 to 1 decimal place
  2.43 rounds to 2.4 as the 3 does not affect the 4
- Round 2.47 to 1 decimal place
  2.47 rounds to 2.5 as the 7 affects the 4

Significant figures – Has to have this number of numbers IN TOTAL but the next number needs to be considered as above and any numbers not used must be replaced with 0's.

- Round 362 to 2 significant figures
  362 rounds to 360 as the 2 does not affect the 6
- Round 368 to 2 significant figures
  368 rounds to 370 as the 8 affects the 6

Multiply and divide fractions

Multiply – multiply the top numbers then multiply the bottom numbers

$\frac{2}{5} \times \frac{3}{8} =$

Divide – Write the first fraction down
Turn the second fraction upside down
Change the sign to multiply
Multiply the 2 fractions

$\frac{2}{5} \div \frac{3}{8} =$

Ratio

Ratio does not have units

Writing ratio and cancelling

- A school has 50 teachers and 900 students
  Write down the teacher : student ratio in its simplest form

  Write the ratio in order ie. 50 : 900 then cancel each by the same number until it will not cancel any further
  Cancelling by 10 gives 5 : 90 then cancel by 5 which gives 1 : 18.

Cancelling ratio where there are units

- Write £3.50 : 50p as a ratio in its lowest terms.

  First make the units the same – 350p : 50p

  Remove the units and cancel – 350 : 50 (by 10)
  35 : 5 (by 5)
  7 : 1

Sharing amounts in a given ratio

- In a school of 1000 students the ratio of boys to girls is 9 : 11
  Calculate the number of boys and girls.

  9 + 11 = 20 so there are 20 'parts'
  1000 ÷ 20 = 50
  Multiply each part of the ratio by this amount
  9 x 50 = 450 boys
  11 x 50 = 550 girls

Unitary method

- A teacher pays £27.60 for 6 calculators
  How much does he pays for 15 of the same calculators?

  Find the price of 1 (£27.60 ÷ 6 = £4.60)
  Multiply by 15 (15 x £4.60 = £69)

Percentage increase/decrease

Increase - find the amount to be increased and **add** to the original amount
Decrease - find the amount to be decreased and **subtract** from the original amount

- Increase 80p by 5% (5 ÷ 100 x 80 = 4, so add 4 to 80 = 84p)
- Decrease 80p by 5% (5 ÷ 100 x 80 = 4, so subtract 4 from 80 = 76p)

Remember that in a non-calculator question, work from 10%

Compound interest

Interest that is calculated based on the amount of the previous year

- I have £2000 to invest in a bank with compound interest rates of 5%. How much will I have at the end of 3 years?

  5% of 2000 = 100 at the end of year 1 I will have 2000 + 100 = £2100
  5% of 2100 = 105 at the end of year2 I will have 2100 + 105 = £2205
  5% of 2205 = 110.25 at the end of year 3 I will have 2205 + 110.25 = £2315.25

Standard form

Large or small numbers that are represented to the power of 10
Large numbers are positive powers and small numbers are negative powers.

The first number must be more than 0 but less than 10
The power of 10 is the number of places that the decimal point moves to effect this.

- $213000000 = 2.13 \times 10^{8}$
- $0.000786 = 7.86 \times 10^{-4}$

When calculating in standard form put the numbers into the calculator using the EXP button:

- $(2.13 \times 10^{5}) \times (3.17 \times 10^{4}) = 6.7521 \times 10^{9}$

  2.13 EXP 5 x 3.17 EXP 4 =

Statistics

TYPES OF AVERAGE

MEAN

The first type of average
Add together all the data
and divide by the total number
of values in the sample

eg 3 5 1 3

$$\text{Mean} = \frac{3 + 5 + 1 + 3}{4}$$

Often the most useful BUT can be badly affected by abnormal values

MEDIAN

Middle value: put the numbers in Order first

eg 1 2 4 3 1

order 1 1 2 3 4
↑
Median

If there are two middle values take the average of those two

Ignores extreme values

MODE

Occurs the most

eg 2 3 3 1 4 3 3 3 1
Mode = 3

RANGE

Shows 'spread' : Highest - lowest

eg. 3 7 1 5 2 1 6
Range = 7 – 1 = 6

STEM AND LEAF

A method of displaying data

eg. 81, 70, 67, 56, 59, 51, 64, 73, 62, 71, 64, 64

1) First decide on the 'stem' for your diagram.
The stem is often the first digit of the numbers
So, we will use the TENS Digit in this example.

5
6
7
8

2) Add the UNITS of each number one by one from the list. DON'T MISS ANY OUT.

5	6 9 1
6	7 4 2 4 4
7	0 3 1
8	1

3) Then sort the leaves into order and add a KEY:

5	1 6 9
6	2 4 4 4 7
7	0 1 3
8	1

KEY: 8 | 1 = 81

MEAN, MEDIAN AND MODE FROM A GROUP

A survey counted the number of eggs in seagulls' nests in June.
These are the results:

Find

a) The mode
b) The median
c) The mean number of eggs.

Number of eggs x	Frequency f	$f \times x$
0	17	0
1	12	12
2	23	46
3	37	111
4	18	72
Totals	**107**	**241**

Mean = $\frac{\sum fx}{x} = \frac{241}{107} = 2.25$

Mode (most) = 3

Median (half way)
Half of 107 = 53.5
54th score = 3

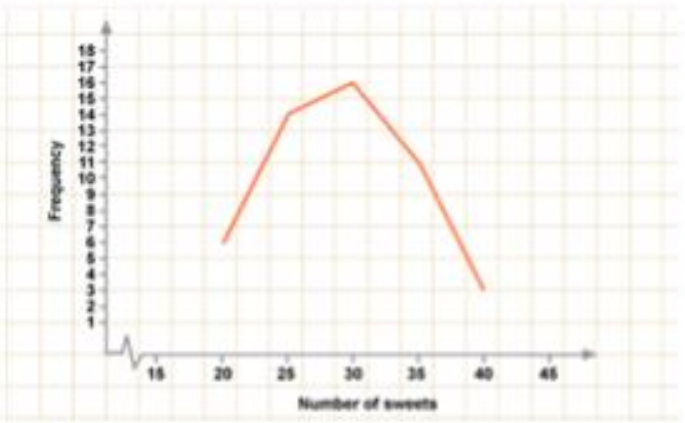
FREQUENCY POLYGON

A Frequency Polygon Uses the Midpoint of Each Class

1) You need to work out the MIDPOINTS of each class of heights first:

Number of sweets	18 ≤ h < 22	22 ≤ h < 27	27 ≤ h < 32	32 ≤ h < 37	37 ≤ h < 42
Midpoint	20	25	30	35	40
Frequency	6	14	16	11	3

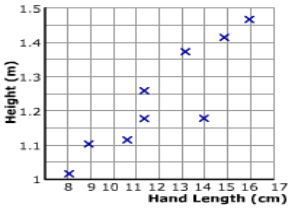
2) Then PLOT the frequency polygon:
Plot the midpoint of each class against its corresponding frequency



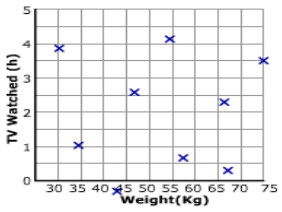
Because it's a polygon (rather than a curve), you join the points with straight lines.
(The straight lines just mean you're assuming the data's evenly spread in each interval – but you don't need to worry about that.)

SCATTER GRAPHS

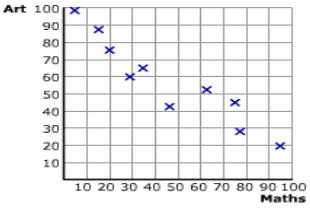
1) A SCATTER GRAPH is just a load of points on a graph that end up in a bit of a mess rather than in a nice line or curve.
2) There's a fancy word to say how much of a mess they're in – it's CORRELATION.
3) Good Correlation (or Strong Correlation) means the points form quite a nice line, and it means the two things are closely related to each other.
4) Poor Correlation (or weak correlation) means the points are all over the place and so there's very little relation between the two things.
5) If the points for a line sloping UPHILL from left to right, then there is POSITIVE CORRELATION which just means that both things increase or decrease together.
6) If the points for a line sloping DOWNHILL from left to right, then there is NEGATIVE CORRELATION, which just means that as one thing increases the other decreases.
7) So when you are describing a scatter graph you have to mention both things, i.e. whether it's strong/weak/moderate correlation and whether it's positive/negative.
8) When reading values off from a scatter diagram, you MUST put in the LINE OF BEST FIT



POSITIVE CORRELATION



NO CORRELATION



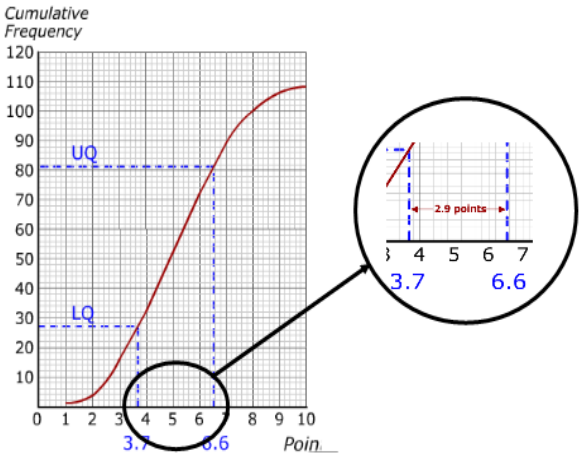
NEGATIVE CORRELATION

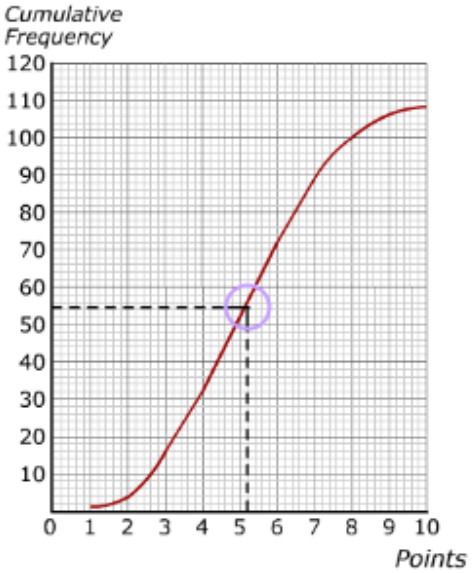
CUMULATIVE FREQUENCY GRAPH

Worked examination question

The grouped frequency table shows the number of points students got in a maths quiz.

Pts	Freq	Cumulative Frequency
1	1	1
2	3	4
3	12	16
4	16	32
5	19	51
6	21	72
7	17	89
8	11	100
9	6	106
10	2	108





QUESTIONNAIRES

There are 3 Main Ways to do Questionnaires

1. FACE TO FACE INTERVIEWS
2. By POST
3. TELEPHONE interviews

These each have their ADVANTAGES and DISADVANTAGES.

1. If you interview face-to-face or by telephone you're more likely to get responses than if people have to post their answers.
2. Also, you can ask more complicated questions, as you can explain what they mean to people who don't understand.

BUT... People are more likely to lie to you in a face-to-face interview and when interviewing by telephone your sample will miss all the people without a telephone (among others).

Make Sure You Don't Collect Biased Data
There are five points to remember here:

1. Make sure you identify the population correctly.
2. Choose ways to distribute and collect your questionnaire that mean as large a proportion of your sample will respond as possible – keep it simple and easy.
3. Follow up people who don't respond to your questionnaire. Don't just forget about them.
4. Make your questionnaire as clear as possible, so people can record their answers accurately.
5. And lastly – don't lose any of your carefully collected data.

Questions Can Be Open or Closed

1. Closed questions have a FIXED NUMBER of possible answers – these could be yes / no or tick box questions.

EXAMPLES:

Are you under 18 years of age? Yes /No

Tick the mode of transport you use to get to school
Bus ☐ Car ☐ Bicycle ☐ Walking ☐ Other ☐

The good thing about closed questions is that you can easily process the data collected
Also, if the question is well designed, the responses won't be ambiguous at all.

2. Open questions allow any answers.

EXAMPLE:
What is your favourite TV program?

Open questions are particularly good in face-to-face interviews because you can follow up the answers. The problem is that you might end up with such a wide range of answers that the data is really hard to process.

Shape

Area

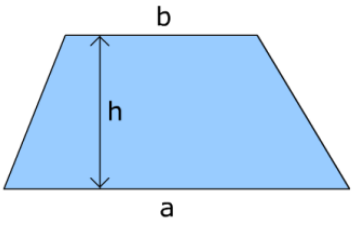
Perimeter

This is the distance around the OUTSIDE of the shape.

Area

This is the amount of space inside the shape.

Formulas



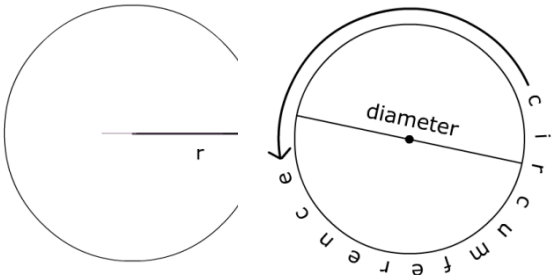
Area of a trapezium = (average of a and b) × h

$$= \frac{(a + b)}{2} \times h$$

Circles

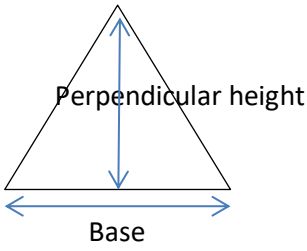
$$\text{Area} = \pi r^2$$

$$C = d \times \pi$$



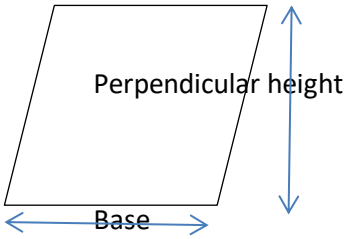
Pi x Radius squared

Triangle



Area = Base x Perpendicular height

Parallelogram

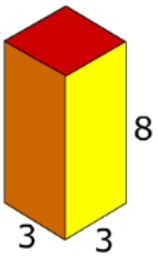


Area = Base x Perpendicular height

Volume

Volume is a measure of the amount of space there is inside a 3-D shape. We measure volume using Cubed (3)

Volme of a cuboid = width x length x heigh



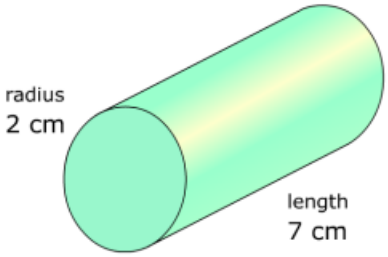
Volume = 3 x 3 x 8

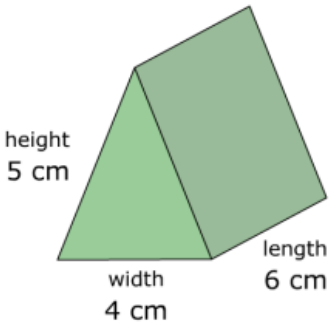
=7 cm^3

Volume of a prism = area of cross section x length

(the cross section will depend upon the type of shape you are dealing with circle, trapezium, triangle or rectangle)

Examples





Find the area of the cross section.
In this case it is a circle.

Area $= \pi r^2$
$= \pi \times 2^2$
$= \pi \times 4$
$= 4\pi \text{ cm}^2$
$= 3{\cdot}14 \times 4$
$= 12{\cdot}56 \text{ cm}^2$

Volume $= 12.56 \text{ cm}^3 \times 7$ slices
$= 87.92 \text{ cm}^3$

Find the area of the cross section.
In this case it's a triangle.

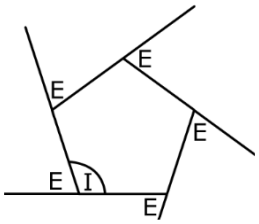
Area $= \frac{1}{2} \times$ base $\times$ height
$= \frac{1}{2} \times 4 \text{ cm} \times 5 \text{ cm}$
$= \frac{1}{2} \times 20 \text{ cm}^2$
$= 10 \text{ cm}^2$

Volume $= 10 \text{ cm}^3 \times 6$ slices
$= 60 \text{ cm}^3$

Angles

Exterior angles always = 360 degrees

(regardless of the number of sides of the polygon)



360 ÷ number of sides of polygon = exterior angle

So therefore the above shape has 5 sides, each of the EXTERIOR angles must be equal = 360 ÷5 = 72 degrees

That means to find the INTERIOR angle

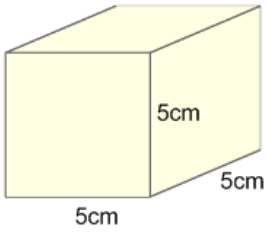
180 – 72 = 108 degrees

The total sum of all the INTERIOR angles = 72 x 5 = 540 degrees (5 sided shape)

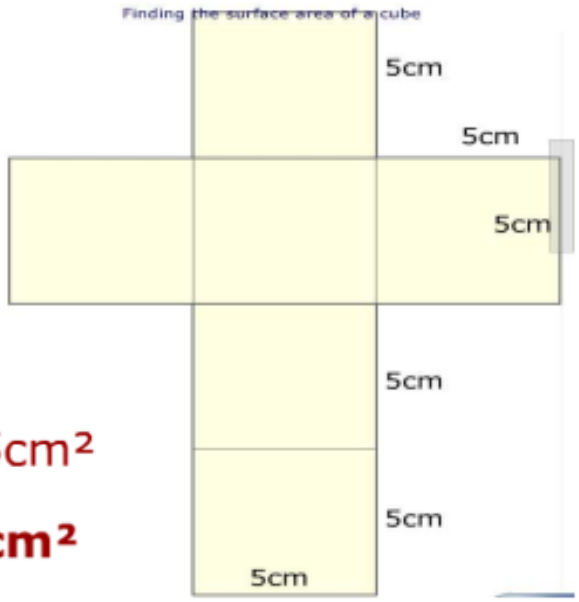
Surface area

To find the surface area of a shape you need to add up the areas of all the faces.

A cube has 6 identical faces.



When you flatten out the cube you end up with its net



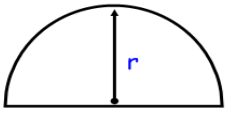
Area of a square = 5 x 5 = 25cm²

Surface Area = 25 x 6 = **150cm²**

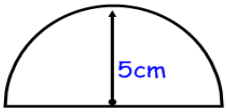
Area and perimeter of a semi circle

Area of a semi circle

Semi circle = half of a circle



area of a semi circle $= \frac{\Pi \times r^2}{2}$



area of a semi circle $= \frac{\Pi \times 5^2}{2}$

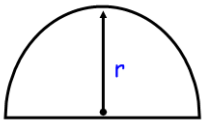
$= \frac{78.5498}{2}$

$=39.269$

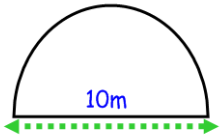
$=39.27cm^2$ (2d.p)

Perimeter of a semi circle

Semi circle = half of a circle



perimeter of a semi circle $= \Pi \times d$



perimeter of a semi circle $= \Pi \times d$

$= \Pi \times 10$

$= \frac{31.41592654}{2}$

$=15.70796327$

$=15.71$ (2d.p)

Transformations - Translation

Translating 2-D shapes using vectors.

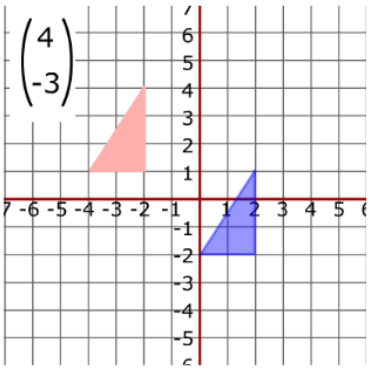
A translation is a movement in a straight line.

For a translation you have to say how far left or right and how far up and down the shape has moved. We use vectors to show how a shape has moved.

Vectors

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ → ↑ $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$ ← ↓

Translate the pink triangle $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$



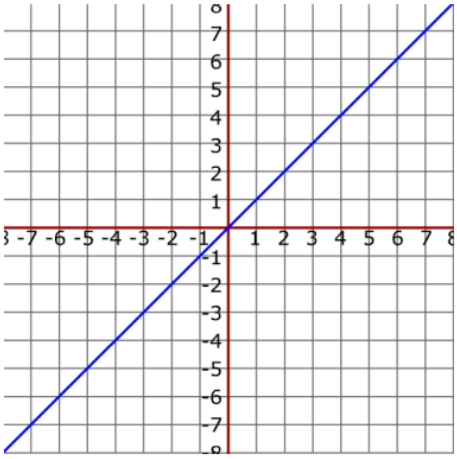
Transformations - Reflection

Reflecting a 2-D shape in the line y = x and y = -x.

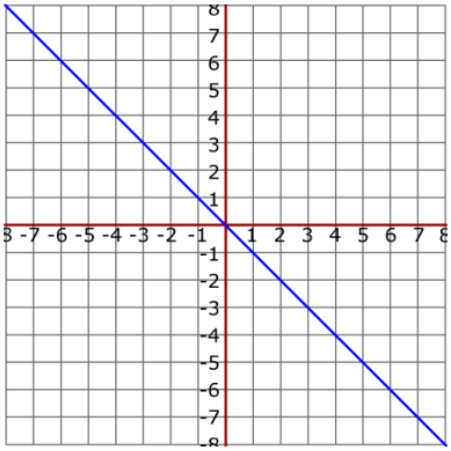
For a reflection you always need to say where the mirror line is.

Sometime you may be asked to reflect in a *diagonal mirror line*. The most common lines are :

Y=x

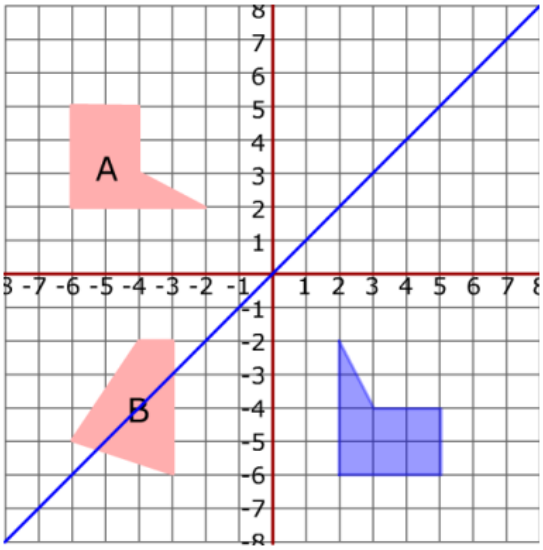


y = - x

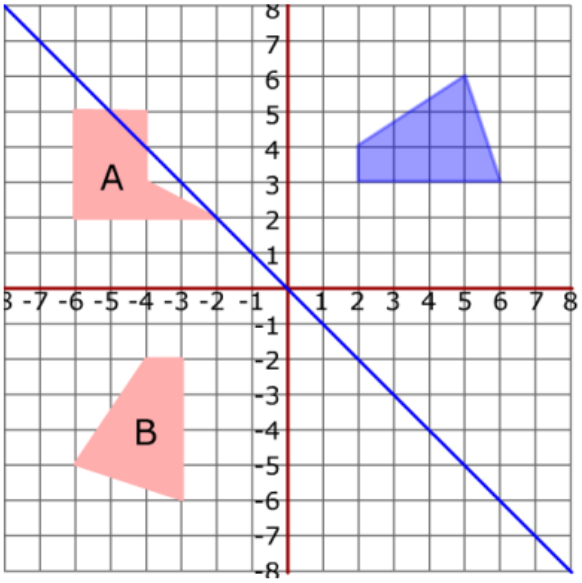


Examples of diagonals

Reflect shape A into the line y = x



Reflect shape B into the line y = -x

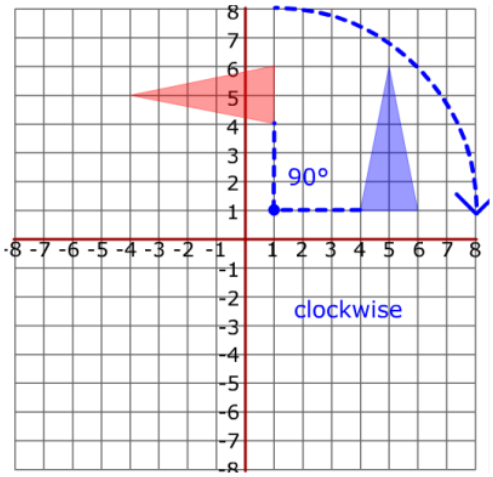


Transformations - Rotation

Rotating a 2-D shape about a point

To rotate a shape you need to be able to say three things

1. Where the centre of rotation is
2. The angle of roation
3. Whether the rotation is clockwise or anticlockwise



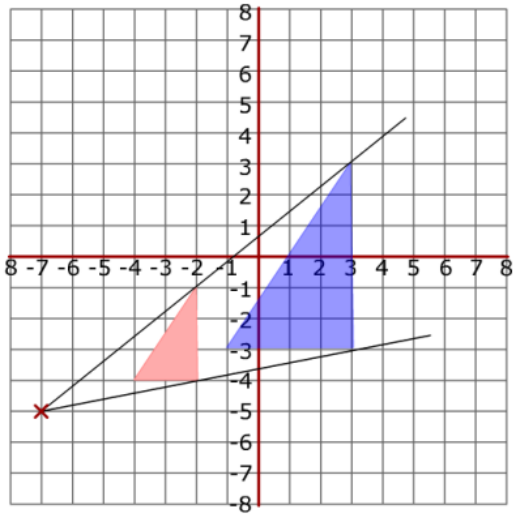
Rotating a 2-D shape about a point

To describe an enlargement you need to give the scale factor and the centre of enlarement.

The SCALE FACTOR of an enlargement tells you how much each length is multiplied by.

Here the Scale factor = x 2

And the centre of enlargement is (-7, -5)



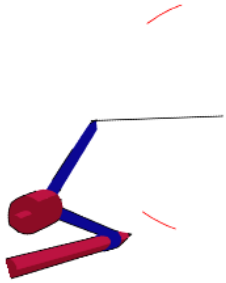
Construction

Constructing a perpendcular bisector

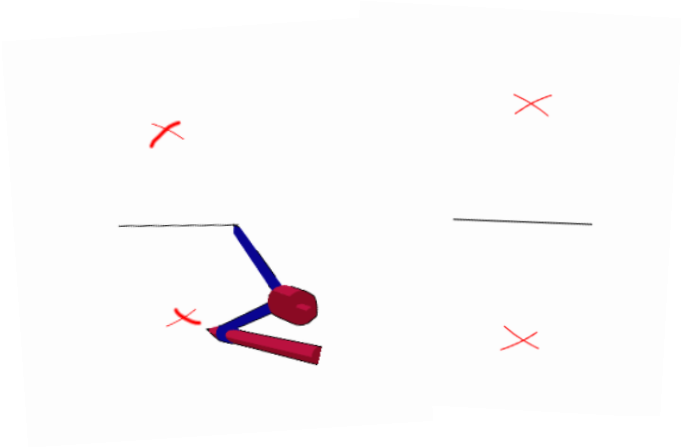
Step 1 Draw a straight line

Step 2 Move your compass to the end of the line

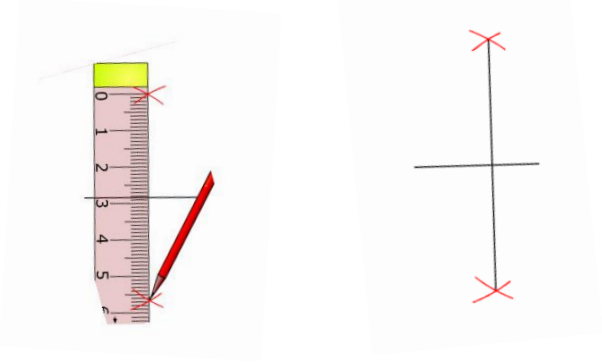
Step 3 Make an arc above and below the line



Step 4 Repeat from the other end of the line

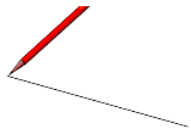


Step 5 Now draw a line through the 2 sets of the arcs

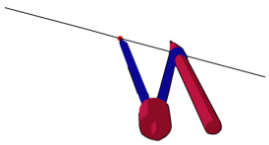


Constructing a perpendicular from any point on a line

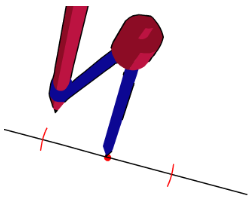
Step 1 Draw a straight line



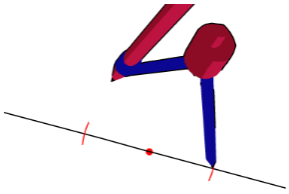
Step 2 Move your compass to the point on the line



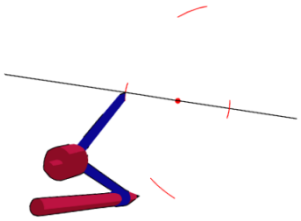
Step 3 Draw an arc on the line either side of the point



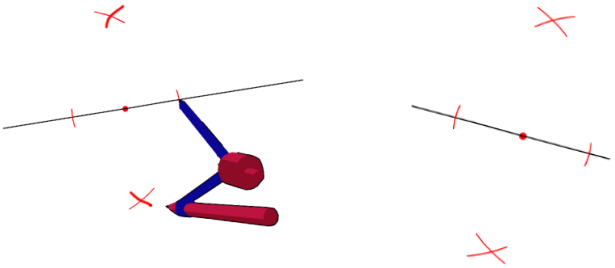
Step 4 Widen the compass and move to an arc



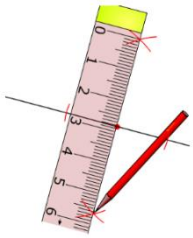
Step 5 Make an arc above and below the line

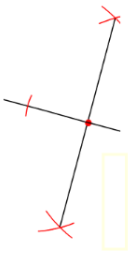


Step 6 Repeat from the other arc



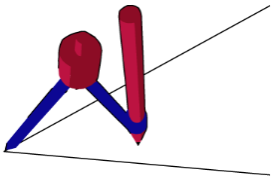
Step 7 Now draw a lin e through the two sets of arcs



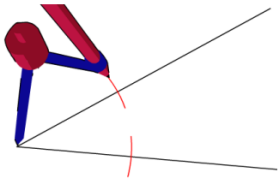


Constructing an angle bisector

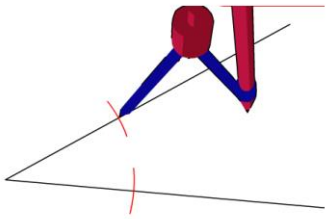
Step 1 Move the compass to the meeting points of the lines



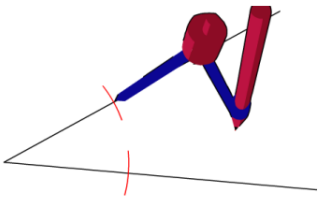
Step 2 Draw an arc on both lines



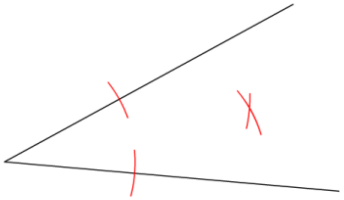
Step 3 Move the compasses to each of the lines in turn



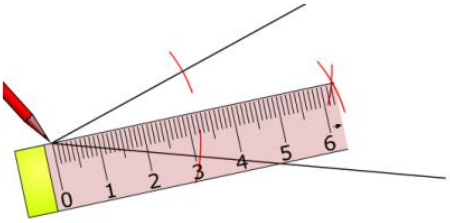
Step 4 Make another arc in the general area of the bisection



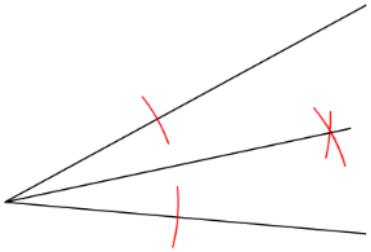
Step 5 Repeat from the other arc



Step 6 Now draw a line from the end of your line through the arcs



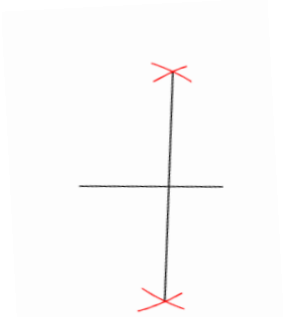
The angle bisector is drawn



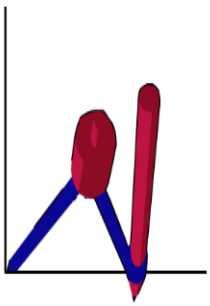
Construct an angle of 45^0

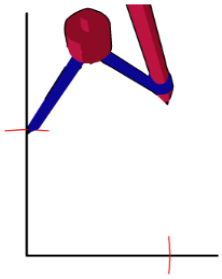
Use a pencil, ruler and compass to construct an angle of 45^0

Step 1 First make a right angle (see constructing a perpendicular bisector)

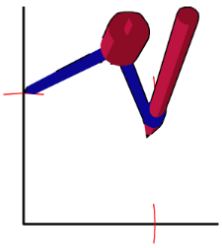


Step 2 Draw and arc on both of the lines

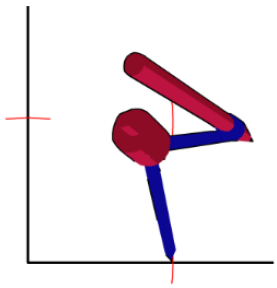


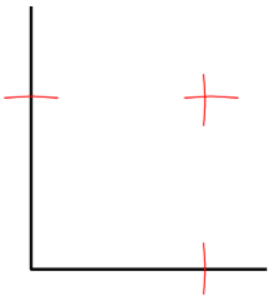


Step 3 Move the compasses to each of the arcs of the lines in turn

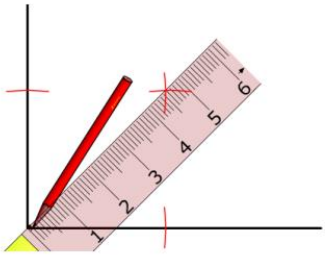


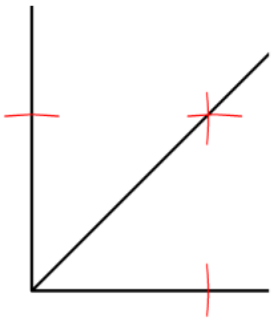
Step 4 Make another arc in the general area of the bisection





Step 5 Now draw a line from the end of your line throughthe arcs





Loci

A LOCUS is a set of points which satisfy a condition.

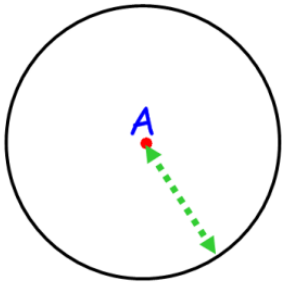
You can construct loci using a ruler and compasses. A set of points can lie inside a region rather than on a line or a curve.

The locus of points which are 7cm from A is the circle, centre A.

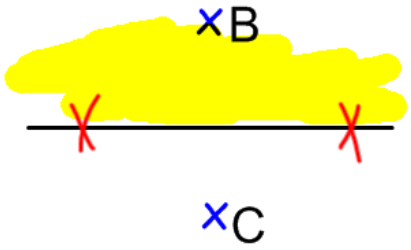
The region of points

Less than 7cm from

A lies inside this circle



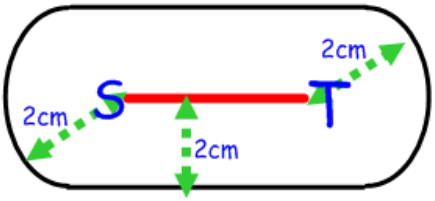
The locus of points which are the SAME DISTANCE from B as from C is the perpendicular bisector of BC



Points in the shaded

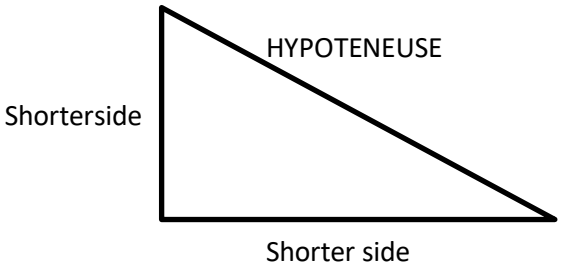
are closer to B than C

The locus of points which are 2cm away from ST consists of two semicircles and two straight lines.

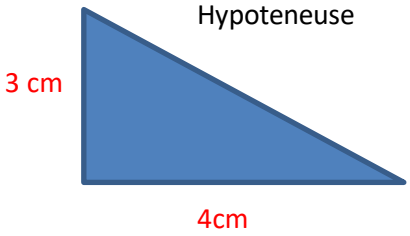


Pythagoras theorem

Pythagoras theorem is used to find the missing side of a right angled triangle



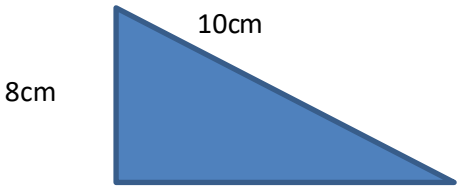
To find the hypoteneuse (the longest side)



1. Square the two shorter sides = 3 x 3 =9 4x4=16
2. Add them together = 9 + 16 = 25
3. Find the square root of the sum = √25 = 5cm

so therefore the hypoteneuse = 5cm

To find the hypoteneuse (the longest side)



1. Square the two shorter sides = 10x10=100 8x8=64
2. Subtract them = 100-64 = 36
3. Find the square root of the sum = √36 = 6cm

so therefore the shorter side = 6cm

Algebra

- Brackets

Single Brackets – Multiply the term on the outside of the bracket by everything on the inside

e.g $2a(a - 5) = 2a \times a + 2a \times -5 = 2a^2 - 10a$

Expand and simplify – Expand the bracket(s) then simplify by collecting like terms
e.g $8(3x + 4) - 2(4x - 5) = 24x + 32 - 8x + 10 = 16x + 42$

Double brackets – Use the grid method or FOIL (Firsts, Outsides, Insides, Lasts)
e.g Expand and simplify $(y - 3)(y + 4)$
Grid method

x	**y**	**-3**
y	y^2	-3y
+4	+4y	-12

$= y^2 - 3y + 4y - 12$
$= y^2 + y - 12$

FOIL Method

F O I L
$(y - 3)(y + 4) = y \times y + y \times 4 + -3 \times y + -3 \times 4$
$= y^2 + 4y - 3y - 12$
$= y^2 + y - 12$

Factorising brackets – this is the opposite of expanding brackets. You need to look for the largest factor you can take out of every term in the expression.
e.g Factorise completely $12x^2 - 18x y$

$6(2x^2 - 3x y)$	6 goes into each term, but this is only partly factorised.
$6x(2x - 3y)$	6 and x go into each term. This expression has been completely factorised.

- Solve equations

Remember to do the same thing to both sides of the equation.
If there's any brackets expand them first.
If there's letters on both sides you need to collect them together on one side.
e.g. Solve $3(x - 4) = 5x + 8$

$3x - 12 = 5x + 8$	Expand the bracket
$-12 = 2x + 8$	Minus 3x from both sides
$-20 = 2x$	Minus 8 from both sides
$-10 = x$	Divide both sides by 2

- Solve inequalities

You can solve an inequality in exactly the same way as you solve an equation.
e.g. Solve $3x + 2 < 5$

$3x < 3$	Minus 2 from both sides
$x < 1$	Divide both sides by 3

Remember this means x is less than 1

- Trial and improvement

  Some equations can't be solved exactly. You will be told when to use trial and improvement in your exam. Draw a table to clearly show your working.

  e.g. The equation $x^3 - 5x = 60$ has a solution between 4 and 5. Use trial and improvement to find this solution. Give your answer correct to 1 decimal place.

x	$x^3 - 5x$	Comment
4.5	68.625	Too big
4.2	52.088	Too small

You know there is a solution between 4 and 5.
x = 4.5 is a good first value to try.
Use your calculator to work out
$4.5^3 - 5 \times 4.5$ and compare your answer with 60.
x = 4.5 is too big

x	$x^3 - 5x$	Comment
4.5	68.625	Too big
4.2	52.088	Too small
4.3	58.007	Too small
4.4	63.184	Too big
4.35	60.56…	Too big

Keep trying different values.
Make sure you write down every trial.
You know the answer is between 4.3 and 4.4. But you don't know which value is closer.
Try 4.35. This will tell you whether the answer is closer to 4.3 or 4.4
4.35 is too big so the answer is between 4.3 and 4.35
Write down the answer correct to 1 decimal place

x = 4.3 (to 1 d.p.)

- Rearrange simple formulae

  Most formulae have one letter on its own on one side of the formula. Changing the subject is just like solving an equation, you have to do the same thing to both sides.

  e.g. $N = \frac{3h+20}{100}$

  Rearrange the formula to make h the subject.

  $N = \frac{3h+20}{100}$ Multiply both sides by 100

  $100N = 3h + 20$ Minus 20 from both sides

  $100N - 20 = 3h$ Divide both sides by 3

  $\frac{100N-20}{3} = h$

  $h = \frac{100N-20}{3}$ It's a good idea to write your answer as h = …

- *n*th term of a linear sequence

  In your exam you may be asked to work out the *n*th term of a sequence. Here are the steps to use.
  The first five terms of an arithmetic sequence are;

  1, 5, 9, 13, 17

  Find, in terms of *n*, an expression for the *n*th of the sequence.

  Step 1 – Write in the difference between each term

  1 5 9 13 17
  +4 +4 +4 +4

  Step 2 – Work backwards to find the zero term

  Zero term
  -3 1 5 9 13 17
  +4 +4 +4 +4

  Step 3 – Write down the formula for the *n*th term

  nth term = difference x n + zero term

  Step 4 – Write the *n*th term for the sequence

  nth term = 4n – 3

- *n*th term of a sequence of powers

  You need to recognise sequences of powers of 2 or 10

  e.g. 2^n = 2, 4, 8, 16, 32, 64, …

  10^n = 10, 100, 1000, 10000, 100000, 1000000, …

- Quadratic graphs

Table of values and curve

You will need to complete, or draw, a table of values. All quadratic graphs have a line of symmetry which passes through a maximum or minimum point.

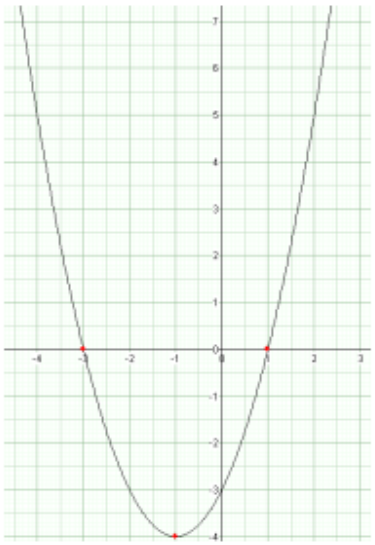
e.g. Fill in the table of values for the equation $y = x^2 + 2x - 3$ and draw the graph

x	-4	-3	-2	-1	0	1	2	3
y	5		-3	-4	-3	0		

Work out each point carefully and then put it in the table

e.g If x = 3, $y = 3^2 + 2x3 - 3 = 9 + 6 - 3 = 12$

Plot the points carefully and join together to make a smooth curve.



Using to solve an equation

e.g. Use your graph to solve the equation $x^2 + 2x - 3 = 0$

Add the line y = 0 to the diagram and read the x-values where the 2 graphs cross.

Remember y = … is a horizontal line.

- Midpoint of a line segment

To find the midpoint between a pair of coordinate find the middle between the x coordinates and the same for the y coordinates.

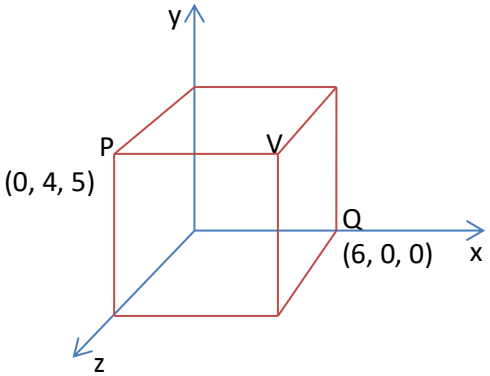
e.g find the midpoint for this pair of coordinates, (-2, 6) and (8, 10)

x -coordinate = $\frac{-2+8}{2} = \frac{6}{2} = 3$ y -coordinate = $\frac{6+10}{2} = \frac{16}{2} = 8$

Midpoint = (3, 8)

- Plot and use 3D coordinates

You can describe a point in three dimensions by using coordinates that have three numbers. Give the coordinates in the order (x, y, z).



The coordinates of V are (6, 4, 5)